

# Implementation of an Effective Height of Bent Thin-Wire in Cartesian FDTD Mesh and Electric Charge Correction on the Staircased Edges

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This paper brings together a set of modifications in the thin-wire formalism for bent wire antennas. In addition it proposes a correction in the charge of the corners formed by the staircase effect of bent thin wires in the FDTD mesh and defines an effective height for the staircased geometry that compensates the delay of the electromagnetic fields propagation. The contributions, that improve antennas characteristics, can be summarized as: 1- projections of the electric and magnetic fields in FDTD mesh, since the fields do not vary in a circular way as in the standard approach; 2- the charges correction on corners that are not included in thin-wire equations; 3- the treatment at the top of the antenna that considers the effect of the charge accumulation in this region. 4- The effective height that is a macroscopic way of adding the charge on the corners since it adjusts the antenna size to consider the low speed of the electromagnetic fields in the staircased wire.

*Index Terms*— Bent thin-wire, Charge on FDTD mesh, Effective Height, Field Projection and End Cap.

## I. INTRODUCTION

The sub-cell model is a method that simulates an effective radius in FDTD without the need of the cell to be smaller than the radius of the wire. In this formulation, the electromagnetic fields are corrected so an infinitesimal wire is equivalent to a wire with the desired radius. In [1] is proposed an improvement in the conventional thin-wire equations presented by Taflove [2] where magnetic fields are projected at the edge of the FDTD cell and the electric field is projected at the cell surface. Also in [1] the correction of the charge at the top of the antenna is made since it is not predicted in the conventional equations.

Holland presented the first approach for oblique thin-wire antennas in the FDTD mesh where two auxiliary equations of current and charges propagation were added [3]. In [4] and [5] it is demonstrated that the thin-wire can be accurately modeled at any location of the Yee cell using cylindrical coordinates. A new oblique thin-wire formalism was proposed in [3] and [6] establishes the wire at any position of the Yee cell. This proposal also uses two auxiliary equations of charge and electric current and makes a treatment for the joints. This approach presents high degree of complexity by changing the FDTD equations.

The contribution of this paper is the implementation of modifications in the thin-wire equations, in order to correct errors in the speed propagation of the electromagnetic fields due to the non-prediction of the accumulated charges in the thin-wire corner and also the delay experienced by the fields due to the longer path to course. In this way it is possible to simulate wire antennas in the conventional FDTD loop without changing the equations for cylindrical coordinates as in [4] and [5], or adding auxiliary equations such as [6].

## II. CHARGES ACCUMULATION ON THE CORNERS

When there are antennas of inclined wire in the FDTD mesh, the electromagnetic wave will decelerate and it will also course a longer path than the real one. This happens due to the zigzag effect [6] of the FDTD mesh. Some factors influence the frequency shift, such as: the antenna has a larger size due to the FDTD stairs effect; the reduction in the speed propagation of the electromagnetic wave due to the stairs effect; charges accumulation in the corners are not predicted by the standard thin-wire formalism; the imprecision of the standard thin-wire formalism in the metal corners since only two electric fields are available in the rotational calculation. In Fig. 1a,  $\Delta$  represents the step of the antenna where  $\Delta = N \cdot \Delta x$  being  $\Delta x$  the mesh discretization. The thin-wire formalism was used in the simulation with  $N = 1, 2$  and  $3$ . It is noted in Fig. 1b that the late-time it is shorter when  $N = 3$ . In this way, the speed of the electromagnetic wave approaches the real velocity improving the frequency shift in the impedance.

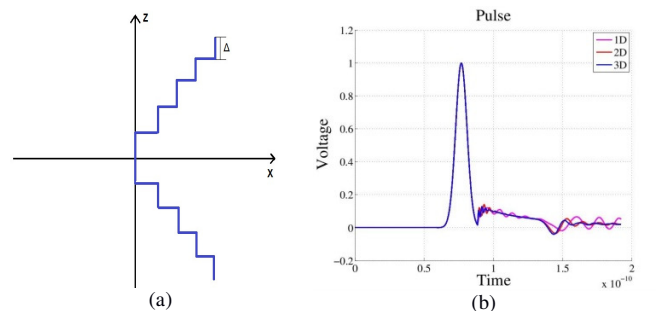


Fig. 1 Vee Dipole in FDTD mesh.

Considering that cubic cells are equal to  $\Delta x = \Delta y = \Delta z = 0.25 \text{ mm}$  and radius of  $0.1 \text{ mm}$ , simulations were

performed with  $N = 1, 2$  and  $3$  and compared with the Method of Moments (MoM) for the staircased-wire geometry and inclined wire geometry, Fig. 1a. Note in Fig. 2 that when  $N$  increases, the result gets closer to the MoM in stairs. This occurs because the speed of the electromagnetic waves propagates with a speed closer to the real one.

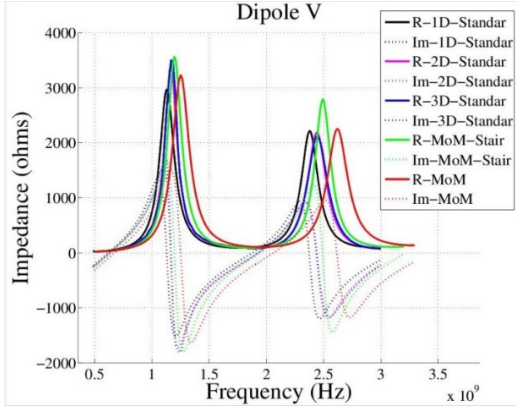


Fig. 2. Comparison of Vee dipoles.

### III. FIELD PROJECTION AND THE END CAP

The electric and magnetic field projections along with the End Cap were applied to the Vee dipole based with  $N = 4$ ,  $\Delta x = \Delta y = \Delta z = 0.4 \text{ mm}$  and a radius equals to  $0.1 \text{ mm}$ . The main functionality of the fields projections is improve the thin-wire equations and the End Cap is to add the charge accumulation at antenna ends which is also not predicted by thin-wire equations [1]. The dashed black line in the Fig. 3 represents the fields designed with the End Cap. This result shows a superiority over the standard equations (without the projections and without the end cap) represented by the blue line. The end cap and filed projections will be a complement of the charger correction; they alone do not solve the frequency shift.

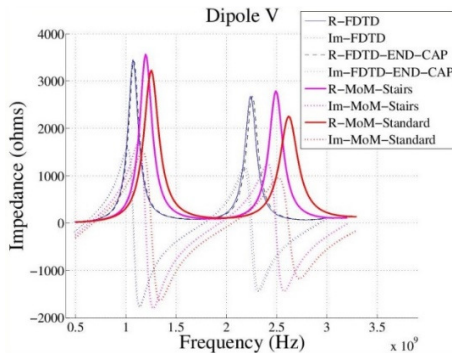


Fig. 3. Vee dipole Standard and Vee dipole with End Cap.

### IV. CHARGE CORRECTION AND EFFECTIVE HEIGHT

The charge accumulation in the corner is added in the standard equations of the thin-wire, since these are not naturally predicted. The charge is calculated in (1):

$$Q_1^{s,n+1/2} = Q_1^{s,n+1/2} + \Delta t \frac{I_1^{s,n} - I_2^{s,n}}{\frac{1}{2}(\Delta x + \Delta y)} \quad (1)$$

where  $I^s$  are the electric currents that pass on the thin-wire and will be calculated by the rotational magnetic fields at  $\Delta/2$  from the corner, where  $\Delta$  is the size of the cell. The charge on the corner is counted on the equations of the electric fields around the wire corner. The complete equation that shows how this charge will be added to the circumferential electric fields to the corner will be available in the 4-page full paper submission.

A macroscopic effect of the charge correction on the corners formed by the FDTD mesh is the effective size of the Vee dipole stems. The effective length is the reduction of the size of the stems so that the electromagnetic wave has the same propagation speed of a wire without corner. In Fig. 4, note the result of the Vee dipole in the FDTD with the effective size compared to the inclined MoM. These considerations will be demonstrated analytically and numerically in the 4-page full paper submission.

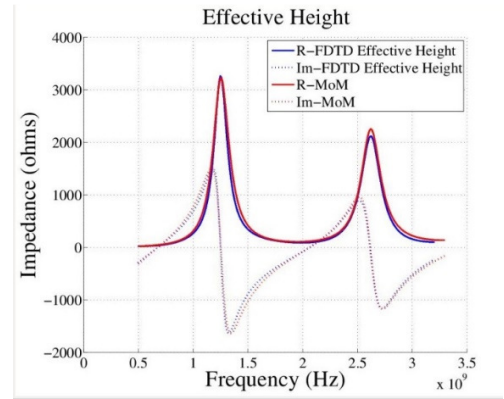


Fig. 4. Vee dipoles with charges in FDTD corners.

### ACKNOWLEDGEMENT

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